

# Some Reflections on Moduli, Their Stabilization and Cosmology

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## Abstract

We review some aspects of moduli in string theory. We argue that one should focus on *approximate moduli spaces*, and that there is evidence that such spaces exist non-perturbatively. We ask what it would mean for string theory to predict low energy supersymmetry. Aspects of two proposed mechanisms for fixing the moduli are discussed, and solutions to certain cosmological problems associated with moduli are proposed.

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# 1 Introduction

Moduli are a crucial ingredient in all of our present understanding of string theory, both at a perturbative and a non-perturbative level. Such particles are not present in nature, and, since string theory otherwise looks a lot like nature, most of us have adopted the view that these moduli will somehow be fixed, or perhaps, in the final description, won't be there at all. Susskind has put forth a provocative, contrary view. He asks: what is strong theory?, and answers: it is that theory which lives on those moduli spaces with enough supersymmetry that we can make definite statements about it, not only perturbatively but non-perturbatively. Such non-perturbative statements include understandings from dualities and, in some cases, non-perturbative formulations (Matrix Theory and the AdS correspondence) at least in some regions of the moduli space (Matrix theory and the AdS correspondence).

This definition goes against what has become the almost conventional wisdom that any quantum theory of gravity is string theory; it is provocative precisely because it excludes the world we see. But it is also provocative because it challenges us to decide: what would constitute a signature for string theory. One example which is often discussed is low energy supersymmetry. In various approximations, low energy supersymmetry emerges naturally from string theory, but it is not at all clear that it is a necessary outcome. This question is brought into sharp focus by recent proposals to solve the hierarchy problem[1, 2, 3], some within a string-theoretic framework, which *do not* invoke low energy supersymmetry. In fact, in the spirit of Susskind's challenge, we can first try to define what we might mean by such a prediction. Supersymmetry, after all, must be a broken symmetry, so what we are asking is whether string theory predicts supersymmetry broken "a little bit," rather than a lot. While we might wish to argue for this based on considerations of hierarchy, it would be far better if we could see that such breaking was intrinsic to string theory. Of course, we might hope to simply solve for some ground state of string theory which has all sorts of desired properties. This state might have some low energy supersymmetry, but (modulo one hopeful comment to appear below) it doesn't seem likely that this will happen soon. Instead, we might try to ask if there is a sense in which low energy supersymmetry might be preferred or generic.

I won't offer an answer to this question, but will try, instead, to state the question in a precise way. What would it mean to predict low energy supersymmetry? After all, how can we tell, without an actual calculation, supersymmetry broken by a small amount from supersymmetry broken by an amount of order one. The key to formulating this question is

the notion of “approximate moduli.” While such a notion falls outside of the strict definition proposed by Susskind, I believe it is quite plausible. Indeed, I would offer two pieces of evidence to suggest we should enlarge Susskind’s definition of string theory. First, there are string vacua with  $N = 1$  supersymmetry which can be constructed perturbatively, and for which one can show that a subspace of the moduli space cannot be lifted non-perturbatively, provided that the theory exists on this space. While one does not have a non-perturbative definition of these states, the strong constraints provided by symmetries and holomorphy are very suggestive[4]. Moreover, these states can be studied both at strong and weak coupling, using Heterotic-Type I and Heterotic-Heterotic string dualities.

Still more interesting, however, are theories where the moduli are lifted. Here one might worry, in the spirit of Susskind’s remarks, that the theories do not exist. But consider the case of large radius compactifications of the heterotic string on Calabi-Yau spaces. At weak coupling the construction is well-known. Classically, there are several moduli. One, called  $S$ , determines the tree level gauge couplings; another set, referred to as  $T$ , determine the size and shape of the internal space. We will caricature this slightly, and call  $T = R^2/\ell_{st}^2$ ,  $S = g_s^{-2}V/\ell_s^6$  ( $\ell_s$  is the string length,  $\sqrt{\alpha'}$ ). In order that string perturbation theory be valid, one needs

$$S \gg 1 \quad S/T^3 \gg 1. \quad (1)$$

One also believes one knows how to describe these states at strong coupling,

$$S \gg 1 \quad T \gg 1 \quad T^3/S \gg 1. \quad (2)$$

Here an eleven dimensional description is appropriate[5]. If  $\rho$  is the size of the eleventh dimension, and  $R$  the size of the internal space,  $T$  now corresponds to  $R^2\rho/\ell_{11}^3$ , while  $S = V/\ell_{11}^6$  ( $\ell_{11}$  is the eleven dimensional Planck length).

Using holomorphy arguments and the  $2\pi$  periodicities of the axions[6], it follows that the gauge coupling functions are of the form

$$f^a = S + n_a T + \mathcal{O}(e^{-S}, e^{-T}). \quad (3)$$

In both the strong and weak coupling regimes, one can have  $S \gg 1$ ,  $T \gg 1$ , so if the theory is consistent, the  $f^a$ ’s should agree in both the strong and weak coupling regimes, and indeed they do[7, 8].

Generically, however, potentials are generated for the moduli in these states. Any superpotential due to short distance string effects is proportional to  $e^{-S}$ ,  $e^{-T}$ . Low energy effects

will generically generate a *far larger* contribution to the superpotential. Gluino condensation is a well-known example, where one generates a superpotential of the form

$$W = e^{(-S-nT)/3b_o}. \quad (4)$$

The corrections to this expression are generally of order  $\mathcal{O}(e^{-2(-S-nT)/3b_o})$ , so for suitably large  $S$  and  $T$ , the superpotential calculation should be reliable, both in the strong and weak coupling regimes. Again, if it makes sense to speak of these states, these expressions must agree between the weak and strong coupling regimes, and they do.

This agreement is strong evidence that these states exist, and that, for large  $S$  and  $T$ , it makes sense to speak of *approximate moduli* and an approximate moduli space. In fact, even the Kahler potentials agree in this region. This can be understood from the fact that, for appropriate  $T$  and  $S$ , in both regimes, the theory is approximately 10-dimensional, with  $N = 1$  supersymmetry. The supersymmetry uniquely determines the form of the ten dimensional lagrangian, up to higher derivative corrections, and this lagrangian, in turn, determines the structure of the leading Kahler potential terms. In other words, there is good reason to enlarge Susskind’s definition at least to a large class of approximate moduli spaces which have  $N = 1$  supersymmetry in various limits. Whether there is a similar class of  $N = 0$  theories is clearly a very important question. As we will discuss later, it is crucially related to the question: does string theory predict low energy supersymmetry.

In this talk, I will focus on the question of stabilization. In particular, I want to consider the possibility that the true vacuum of string theory lies on such an approximate moduli space. This has been the viewpoint in virtually all thinking about string phenomenology, both because it is the best we can do at the moment, but also because it is consistent with some basic facts of nature. It need not hold. But I wish to ask here two questions:

- What sorts of stabilization mechanisms might explain this fact, for particular approximate moduli spaces (note this replaces the usual phrase, “string vacua”). I will focus particularly on the “racetrack mechanism” as a possible explanation[9, 10, 12, 11].
- Can we phrase any the problem of obtaining generic string predictions in this language.

Let me focus on the second question first, and consider, in particular, the question of low energy supersymmetry as a string theory prediction. In the past, we have usually spoken about “classical solutions which respect  $N=1$  supersymmetry.” But we are probably not interested in classical solutions. So we need a more precise definition. If supersymmetry is broken, what

distinguishes, say, the moduli spaces of heterotic string compactifications on Calabi-Yau, from those which, to leading order in coupling, have no supersymmetry? The best formulation which I have been able to come up with is the following. We can distinguish two classes of moduli spaces without supersymmetry:

- Those in which supersymmetry is restored in certain regions of the moduli space, and where there are only a finite number of light states (compared to some characteristic energy scale) in some of these regions. In particular, in these regimes, there is only one light spin 3/2 particle. These we will refer to as approximate string moduli spaces with low energy supersymmetry, or LES.
- Those in which supersymmetry is restored only in regions where there are an infinite number of light states. In particular, in these regimes, there are an infinite number of light spin 3/2 particles. These we will describe as states with bulk susy. In general, this bulk supersymmetry need not manifest itself in low energy physics in any conventional sense. Rather than light spin-3/2 particles, there may be light spin-1/2 particles associated with supersymmetry breaking by branes. The phenomenology of these is not generic.
- Moduli spaces in which supersymmetry is restored nowhere in the moduli space. We will refer to these simply as non-supersymmetric strings.

Calabi-Yau spaces, both at weak and strong coupling, are examples of the first. As the heterotic dilaton tends to infinity, supersymmetry is restored, and there are only a finite number of states which are light compared to the string scale. One *can* take moduli to infinity (the “T” moduli) so as to obtain infinite numbers of states. The second are exemplified, for example, by Rohm type compactifications[15], where light states only appear as some radius tends to infinity; in this limit, there are an infinite number of Kaluza-Klein states, and in particular an infinite number of gravitinos. Examples of non-supersymmetric (approximate) moduli spaces are provided by compactification of the ten dimensional  $O(16) \times O(16)$  string.

Recent proposals for very large dimensions can be discussed in this framework. In particular, it has been argued that a low string scale might explain the hierarchy without supersymmetry. At the same time, most of these proposals invoke some degree of bulk supersymmetry to resolve various questions, and in particular to explain the size of the internal dimension without invoking large numbers[16]. One of the nicest proposals is that supersymmetry is preserved in the bulk and on “our” brane, while being broken on some distant brane[17]. Such a proposal

is in the second class, even though its phenomenology resembles that of conventional supergravity models. (The detailed phenomenology is different, since there is no scale at which the conventional  $N = 1$  supergravity lagrangian is appropriate, so it is probably possible, at least in principle, to distinguish these possibilities.) The third possibility might be illustrated by a model of the Randall-Sundrum type. The authors of [18] have argued that supersymmetry is not necessary in this case to explain the large hierarchy. Whether this solution is really stable, for example, to perturbations of the action used in these analyses (e.g. addition of  $\mathcal{R}^2$  terms to the action) is a question which is currently under study, though at the moment the answer seems to be yes.[19].

This, then, is what one might mean by the statement that string theory predicts low energy supersymmetry: string theory predicts that we sit in an approximate moduli space of the LES type. Of course, we have not established that string theory makes such a prediction, but at least we have succeeded (if somewhat crudely) in stating what the question is. There is certainly a prejudice that the non-supersymmetric states may not exist. They suffer tachyons and other instabilities, and it is hard to see how one would set up any non-perturbative formulation[20], but there is certainly no hard argument that either the LES's make sense (and in particular, there is no calculation which shows that the moduli are ever stabilized in on such a space), or that the non-supersymmetric states do not.

One, albeit limited, approach to this problem is the following. One can go to points in these moduli spaces with discrete symmetries, and ask whether or not these symmetries are anomalous. Such an anomaly would almost certainly signal an inconsistency. Searches among LES models have failed to yield such anomalies. This appears non-trivial. In many case, anomaly cancellation is through a Green-Schwarz mechanism. Currently, a search of moduli spaces with bulk susy is in progress. If one found examples of theories with anomalies, this would suggest that, generically, such models are inconsistent.

## 2 Kahler Stabilization and The Racetrack Models

In thinking about stabilization, there are some basic issues to keep in mind. Given the statement that string theory is a theory without parameters, one would expect that stabilization should occur in regimes where all of the couplings are strong. There is nothing, in principle, wrong with this possibility. It is disappointing in that it could mean that nothing is accessible to calculation, and that any real predictions from string theory are beyond reach. On the other

hand, if we believe that string theory has anything to do with nature, we need to explain the fact that the gauge couplings are weak. Indeed, this might make us optimistic that some quantities, in the end, will be calculable. Another source for optimism is the existence of hierarchies: this also suggests that there should be small parameters in the problem. Our earlier discussion of approximate moduli spaces suggests that hierarchies be explained by stabilization on such a space, where the couplings are small, and  $e^{-S}$  and  $e^{-T}$  are tiny. Non-supersymmetric proposals involving large dimensions also rely on such small factors[16, 18]. Of course, these facts could be accidents. It could be, for example, that a strong coupling theory accidentally predicts a number of order  $1/30$ , and the hierarchy is the result of low energy physics. In this case, again, it would be hard to make predictions from the underlying theory; indeed, it would be hard to argue (in the spirit of Susskind's challenge) in what sense this theory was connected with anything we call string theory.

The focus, then, of any attempt to understand the fate of the moduli should be on the questions:

- Why are the gauge couplings small (and unified)?
- Why are there large hierarchies?
- Can anything be calculated, i.e. is it possible to relate the phenomena to the underlying fundamental theory?

Within the framework of the LES models, there have been three proposals for stabilization of the moduli. The first is known as “Kahler Stabilization”[6]. Here one assumes that the moduli (say  $S$ , the modulus which determines the gauge couplings) indeed are large, so that holomorphic quantities are given by their forms in the large  $S$  limit (weak coupling string, or supergravity limit, say); on the other hand, one supposes that the Kahler potential is far from its weak coupling form, and is such that it leads to stabilization. In terms of the picture of the moduli space which we have described for the heterotic string this seems quite reasonable. There are certainly regions of this moduli space where holomorphic quantities are calculable, but the Kahler potential is not. It is possible, then, that the superpotential might be understood as a result of gluino condensation, while the Kahler potential would be quite different from its very weak or very strong coupling form. One can easily write down Kahler potentials which stabilize the moduli, and, at the price of a mysterious fine tuning, give vanishing cosmological constant. If this picture is correct, holomorphic quantities can be calculated by passing to one or the other limit, but non-holomorphic quantities are not calculable.

The second scenario which has been widely considered in the literature is known as the racetrack model[9, 10]. The racetrack models illustrate several possibilities for stabilizing the moduli:

- Stabilization at scales of order one (with supersymmetry broken or unbroken), and with nothing calculable.
- Stabilization, at the price of discrete fine tuning, with a hierarchy of scales, and at least some quantities calculable.

The racetrack idea is a very interesting proposal for stabilization, which has existed for a number of years in a sort of limbo, because of two issues. First, there are no compelling models in weakly coupled string theory for the phenomenon. Second, it was hard to see why supersymmetry would be broken in a suitable way (and certainly not with vanishing cosmological constant). Finally, and perhaps most fundamentally, given that one was considering a theory without any parameters, how could there really be a scheme in which one could hope to calculate anything?

Kaplunovsky and Louis, however, revisited some of these issues a few years ago, in light of duality and in particular in light of developments connected with F theory[10]. They pointed out that it is easy to obtain huge groups in string theory, so constructing models may not be such a problem. They did not, however, really address the question of what might be computable in such a scheme. Their ideas, however, illustrate many of the main points.

In the racetrack model, one imagines one has two or more strongly interacting gauge groups, and analyzes the problem from the perspective of the low energy theory. We have already argued that such a low energy analysis is often appropriate. Suppose, for example, that one has several gauge groups in the low energy theory without matter fields. Then it is usually argued that the superpotential has the form (in the notation of [10]):

$$W = M_p^3 \sum_a C_a(T) e^{-\frac{6\pi}{b_a \alpha_a(T)}}. \quad (5)$$

Kaplunovsky and Louis then argue that we now know compactifications for which the  $b_a$ 's are enormous, and that generic stationary points of the potential may well have  $6\pi/\alpha \sim b_a$ , and that this is a way to generate a small gauge coupling.

There are, however, some problems with this argument:



- In general, as noted by these authors, the scale of the “low energy theory,” in these cases, is of order  $M_p$ , i.e.  $e^{-\frac{2\pi}{b_a \alpha_a T}} \sim 1$ . These authors then argue that this is a mechanism which can produce a small coupling, but that because the scale is large, one must suppose that supersymmetry is unbroken by this superpotential. However, the problem is deeper. The low energy analysis is simply inconsistent in this case. There is no sense in which there is any small parameter which might justify the analysis. So, while this may provide a toy model for how small couplings might be generated, it does not explain why anything would be calculable, and there is no hope that it corresponds to any sort of reliable analysis one could hope to do in some limit of string theory.
- In general, it is not possible to argue that this is the correct form of the superpotential at general points in the moduli space, even for very small values of all of the couplings. The problem is that the usual sorts of symmetry arguments or dynamical arguments which are made for the superpotential do not generalize straightforwardly in the case of several gauge groups. It is, however, possible to argue that the stationary points one finds in this way may indeed be stationary points of the true low energy effective action. These issues will be discussed elsewhere.

As noted by the authors of [10] (and was an essential feature of the original racetrack proposals), a discrete fine tuning can give a hierarchical ratio of scales, and here we see that this is essential if there is to be *any* sense in which quantities may be calculated. To simplify the notation, call  $b_i = 3N_i$ , and suppose that there are two groups with very similar  $\beta$  functions. Then the stationary point for  $S$  is

$$S = \frac{N_1 N_2}{N_1 - N_2} \ln\left(\frac{C_2 N_1}{C_1 N_2}\right) \quad (6)$$

If  $N_1 \approx N_2$ , this is of order  $1/N^2$ .<sup>1</sup> In this case, the exponential is  $e^{-1/N}$ , and thus is hierarchically small. As a result, the superpotential and gauge coupling functions receive only exponentially small corrections to their values for very large  $S$ . So there is some hope that *holomorphic* quantities are calculable. Inherently stringy effects are of order  $e^{-N^2}$ , so unless they have enormous coefficients, they are negligible.

One can now ask: are non-holomorphic quantities calculable? One might expect that the answer is no, for the following reason. Suppose that one calculates, for example, corrections to

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<sup>1</sup>Note that it is important that  $C_1/C_2$  not be too close to one, or  $S$  will not be sufficiently large. In the model of ref. [12] which we will discuss below, the  $C_i$ 's involve independent couplings, so there is no reason that the ratio should be close to one.

the Kahler potential for  $S$ . In the heterotic string, these are down by  $g^2$ , but there are loops with  $N^2$  particles, and  $g^2 \sim N^2$ . This was one of the points made in [11]. However, there are some possible loopholes to this argument. First, other moduli may appear. In the case of the heterotic string, loop diagrams may involve extra factors of  $1/T$ . To see this, consider the low energy effective theory, and consider the usual form of the weak coupling effective action. In this action, the kinetic term for  $S$  has a factor of  $1/S^2$  out front. The one loop term has two factors of  $S^{-1}$  from propagators, but also a factor of  $\text{cutoff}^2/M_p^2$ . Here the cutoff should be the string scale, which is related to the Planck scale by a factor of  $S$ . So there is no factor of  $T$  here. But in the strongly coupled heterotic theory, the story is different. The cutoff is presumably the eleven-dimensional scale, which is related to the Planck scale by a factor of  $ST$ . So one gets an additional factor of  $T^{-1}$  in the result. So if  $T$  is somehow large as well, it is possible that the perturbation expansion might make sense.

For the Type I theory, the situation is different again. Here, the loops contributing to the  $S$  Kahler potential come with factors of  $g_s^2 N^2$ , but  $g_s \sim g_{YM}^2$ , so now loops are in some sense suppressed by  $1/N^2$ . (I thank I. Antoniadis for some remarks which prompted this discussion.)

Of course, in both the strongly coupled heterotic and the Type I case, one does not know how to construct vacua with arbitrarily large  $N$ , so it is not clear whether such a systematic calculation is possible. But perhaps we can be optimists. After all, we might imagine that for suitably small (but not infinitesimal) couplings, the calculation of the low energy effective action at the cutoff scale is reliable in string theory. We might then expect that the low energy loop corrections are also small for these values of the couplings. So perhaps, under some circumstances, the racetrack picture might provide a possibility of understanding not only why couplings are small, but why both holomorphic and non-holomorphic quantities might be calculable. Of course, there will be many other questions to understand, most urgently the cosmological constant.

Let us assume, for the moment, that only holomorphic quantities can be reliably calculated. Then we might want to limit our attention to theories where supersymmetry is unbroken at the minimum, with vanishing cosmological constant, which means

$$\frac{\partial W}{\partial \phi} = W = 0. \tag{7}$$

This can occur in theories with discrete  $R$  symmetries. A model (with continuous  $R$  symmetries, which can be generalized trivially to the case of discrete  $R$  symmetries) was suggested in [12]. The model is somewhat complicated, requiring many scalar fields. Still, it provides an existence

proof that such a stabilization is possible, at least in principle. In such a theory, we might well hope to calculate holomorphic quantities. For these, the expansion parameter would be  $e^{-1/N}$ , and this is quite plausibly small enough. Non-holomorphic quantities, we have seen, might also be calculable, depending on whether  $1/N$  can be thought of as a small expansion parameter. We have argued that this might be wishful thinking, but it is not totally implausible.

If supersymmetry is broken at the minimum, there are many issues which must be dealt with. First, it is less clear that the superpotential used at the minimum is reliable; one needs to rely on  $1/N$  as a small parameter to assess this. Said another way, if there are large corrections to the Kahler potential, they can effect the location of the minimum. One will have to face more directly, in this case, the problem of the cosmological constant, even in determining the location of the minimum. Still, the fact that, in some sense, even Kahler potential corrections *might* be calculable in some circumstances is an intriguing one.

### 3 Some Cosmological Questions

There are a number of ways in which moduli are likely to be relevant to cosmology. One might imagine that they could act as inflatons[22]; this view has been persuasively put forth recently by Banks[23]. In the rest of this section, we pursue some of the cosmological issues associated with the Kahler stabilization and racetrack scenarios.

A number of cosmological difficulties related to moduli have been considered in string theory. One is the cosmological moduli problem[24]. A number of solutions have been proposed to this problem and I will not review them systematically here. Let me mention just two possibilities. First, it could be that the moduli are heavier than naively expected, i.e. 10's of TeV[24]. Then their decays restart nucleosynthesis. In general, one might worry that it is difficult in such a scheme to produce baryons, but the authors of [21] have argued that in the presence of an Affleck-Dine condensate, there is no problem. An alternative is that the minima of the moduli potential are at points of enhanced symmetry. The usual problem with this is that one might expect any enhanced symmetry point for the dilaton (that modulus which determines the values of the observed gauge couplings) to lie at  $\alpha \sim 1$ . Some scenarios where this might not hold have been considered, but the racetrack picture suggests another possibility[13]. Perhaps this coupling is fixed by this mechanism, in such a way that supersymmetry is unbroken and this modulus is very massive. The other moduli could then sit at enhanced symmetry points, and the universe could naturally find themselves sitting at such a point.

This then raises the question of whether the dilaton itself, even if massive, is likely to settle into its minimum, and this is the question I would like to focus on in the rest of this talk. This problem was discussed by Brustein and Steinhardt some time ago[25] (though I understand that Peskin et al were aware of this issue). They observed that if the superpotential really has the structure suggested by gluino condensation (say with stabilization as in the racetrack scheme, or through Kahler stabilization), it is difficult for the system to find its true vacuum. The point is readily illustrated if we use the Kahler potential suggested by the weakly coupled theory. Then the canonical field (say corresponding to  $S$ ) has the form:

$$S = e^\phi. \tag{8}$$

So the potential behaves roughly as

$$e^{-1/N} e^\phi. \tag{9}$$

This is extremely steep, and the difficulty is that the system is likely, even if it starts at larger coupling than the coupling at the minimum, to overshoot the minimum.

One might object that this argument relies on using the weak coupling form of the Kahler potential, while we have argued that, even though the superpotential might be similar to that expected from the weak coupling analysis, the Kahler potential is likely to receive large corrections. Nir, Shadmi, Shirman and I have looked at this problem, however, and found that modifications of the Kahler potential can only solve the problem if one does drastic fine tuning. If one only fine tunes the Kahler potential and its first and second derivatives near the minimum, this is not enough. Essentially, the Kahler potential must be fine-tuned over a finite range in field space around the minimum.

A second possible resolution of this problem is provided by observations in [14]. These authors note that it is not consistent to suppose that zero-momentum moduli dominate the energy density at early times. The point is easy to understand. Consider the case of the weak coupling dilaton again. Assume that the energy density is dominated by  $\phi$ . In any regime where the potential can be neglected, the equation of state is simply  $p = \rho$ , so the scale factor grows as  $R(t) \sim t^{1/3}$ . As a result, the canonical field obeys the equation:

$$\ddot{\phi} + \frac{1}{t}\dot{\phi} = 0, \tag{10}$$

with solution

$$\dot{\phi} = \frac{c}{t} \quad \phi = \ln t + d. \tag{11}$$

From this equation, it is clear that the potential falls off much more rapidly with time than the kinetic terms; this is the Brustein-Steinhardt problem again. Now consider, however, the equation for the non-zero momentum modes. This is

$$\ddot{\phi} + \frac{1}{t}\dot{\phi} + \frac{k^2}{R^2}\phi = 0 \quad (12)$$

with solution

$$\phi = \frac{1}{t^{1/3}} \cos(3/2)t^{2/3}. \quad (13)$$

As a result, the energy density of the non-zero modes falls off as  $t^{4/3} \sim R^4$ , i.e. more slowly than that of the zero modes (which falls off as  $1/t^2$ ), and just like that of radiation.

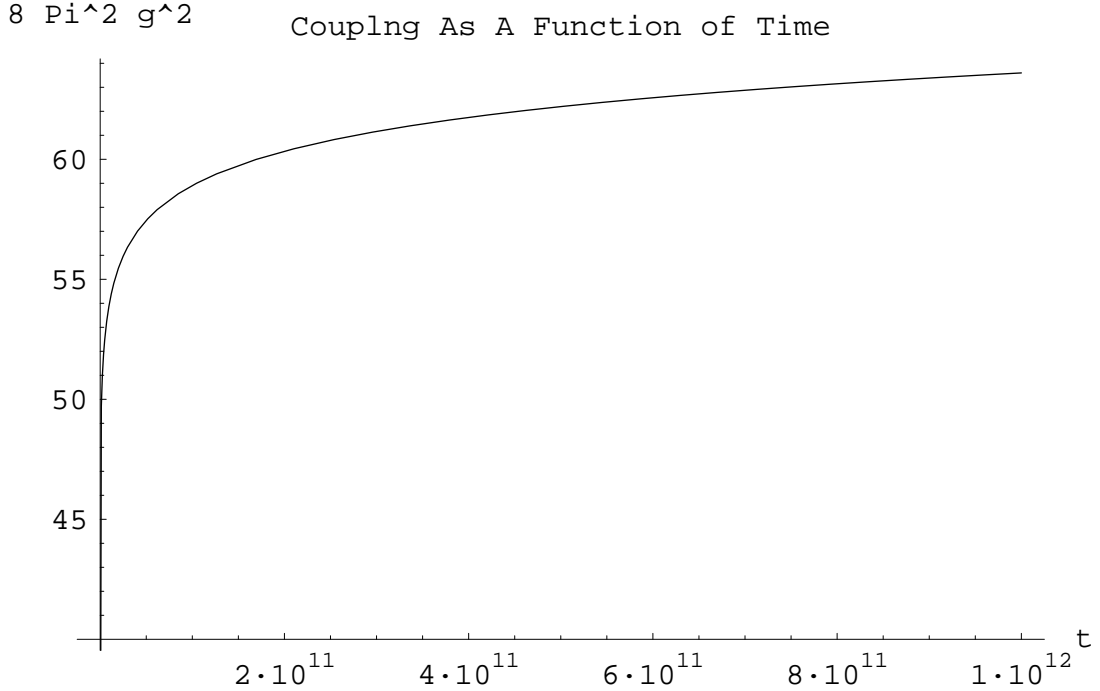


Figure 1: Coupling as a function of time in a radiation dominated universe.

So, while we do not really understand what might be appropriate initial conditions for this system, it is clear that assuming that the field is homogeneous on some scale is not self consistent. Because non-zero momentum modes behave like radiation (e.g. assuming isotropy,  $p = 1/3\rho$ ), one way to model this system is by supposing that the universe is radiation dominated. Note that this does not require that the system be in thermal equilibrium; simply that the non-zero modes of the moduli dominate the energy density, and are roughly isotropic. In this case, the equations of the problem are different. In particular, if one can neglect the potential, the zero

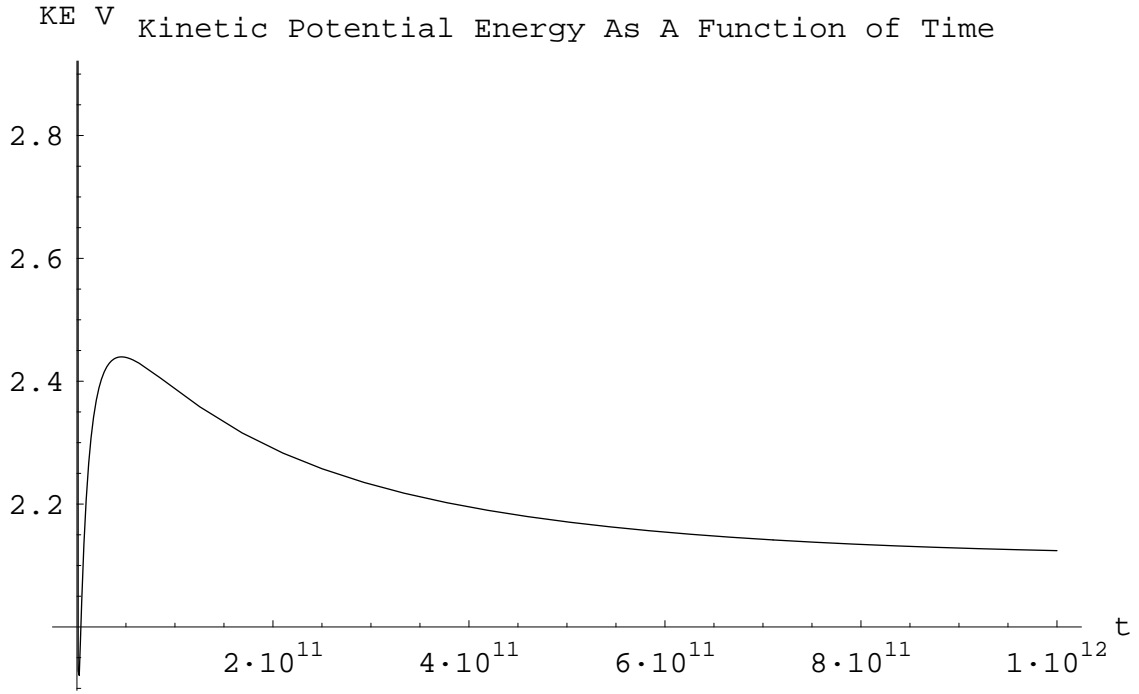


Figure 2: Ratio of kinetic to potential energy.

mode now obeys the equation

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} = 0. \quad (14)$$

This has solution

$$\phi = at^{-1/2} + b. \quad (15)$$

In other words, the field creeps to some particular point. Including the potential, it is now reasonable to hope that the system will track the potential, and eventually settle into the correct minima. Numerical study indicates that this does indeed occur for a range of initial conditions. This can be seen in figs. [1,2]. In these figures, the evolution of the coupling is plotted in a purely exponential potential. One sees that the motion of the field is very slow; moreover, its kinetic energy is never much larger than its potential energy. As a result, when the field reaches the minimum, it does not overshoot.

There is, however, a difficulty with this picture. While the energy may be dominated by the kinetic terms of some field(s), these themselves will receive  $g^2$ -dependent corrections. These corrections may well be much larger at the relevant times than the non-perturbative corrections to the potential, and may shift the location of the minimum. In this case, one must make some

assumption about these corrections and follow more carefully the evolution. In particular, the potential might well force the system to weak coupling and away from the desired minimum. In this case, the Brustein-Steinhardt problem is replaced by an even more severe difficulty. On the other hand, if the potential has a local minimum at relatively strong coupling, a gentle landing is still possible. This problem will be described in detail elsewhere, but its essential features are modeled by the situation where the system is truly in thermal equilibrium, which we turn to now.

An alternative possibility is that the system is truly in thermal equilibrium, i.e. that the gauge bosons, etc., are all in thermal equilibrium. In this case, the analysis required is quite different. For all but very large values of the coupling, the zero temperature dilaton potential is irrelevant; the largest contribution comes from the coupling-dependence of the free energy. In particular, provided  $T \gg \Lambda(S)$ , we can simply take over the high temperature expression for the free energy. This has the form

$$V(g^2, T) = T^4(-a + bg^2 + cg^3 + \dots). \quad (16)$$

If one examines the explicit form for the coefficients  $a, b, c$  in the case of  $SU(N)$ [26], one finds that for  $N = 10$  the coefficient  $c$  is greater than  $b$  for  $g \sim 1$ , so the perturbation expansion is already not reliable for rather modest  $N$ .

One can then imagine several possible behaviors for the effective potential as a function of  $g$ . It might have no minimum, simply falling to zero at small coupling. In this case, the system will not find the true minimum. It might have a local minimum at some  $g = g_o(T)$ , where given that the potential for large coupling ( $\Lambda > T$ ) grows so steeply,  $g_o(T)$  decreases with temperature. In this case, one might expect that the system will roughly track the minimum, and that it will end up being set rather gently in the true minimum. We have checked that this does occur for a range of initial conditions.

To summarize, the extent to which the Brustein-Steinhardt problem is a problems seems quite sensitive to the initial conditions, which are certainly not well understood at present.

## 4 Conclusions

The question “what is string theory,” has been given new focus by recent proposed solutions to the hierarchy problem. Susskind has argued that the only aspects of this theory which we really understand are those associated with states with a high degree of supersymmetry. This

is troubling, as he notes, because these states do not resemble the world we observe. I have argued that we can, with some confidence, extend this list at least to certain exact moduli spaces with four supersymmetries, and to *approximate moduli spaces* which, in certain limits, have four supersymmetries. Two facts of nature suggest that the world we see might sit on such a space: the smallness of the gauge couplings and the existence of hierarchies. While generically one might expect that couplings should be of order one and all scales in the theory should be comparable, we have reviewed at least two plausible mechanisms for fixing the moduli on such a space. We have seen that some – and conceivably all – quantities might be calculable in this circumstance.

This discussion has also allowed us to frame certain issues related to proposals for a low string scale as a solution to the hierarchy problem. We have asked what it would mean for string theory to predict low energy supersymmetry, and argued that this would correspond to demonstrating that the ground state of the theory should lie on an approximate moduli space with  $N=1$  supersymmetry in the sense described above. The low string scale proposals are generally not in this class. In order to explain a large hierarchy of scales, most of these proposals invoke supersymmetry in the bulk. But exact supersymmetry is recovered, if at all, only in the limit where the extra dimensions become infinitely large, so that (from a four-dimensional perspective) there are an infinite number of gravitinos. Interestingly, the proposal of [17] has a low energy phenomenology much like that of the  $N=1$  models. It has been argued[18] that in the Randall-Sundrum picture, supersymmetry might not be necessary even in the bulk to understand the hierarchy. It would be interesting to investigate this statement carefully, and in particular to make sure that the results found in a classical analysis are not spoiled by quantum effects.

The real question, however, is whether there is any way to understand why approximate moduli spaces with low energy supersymmetry should or should not, in any sense be favored by string dynamics. There has long been a view that this is the case, but it is largely based on prejudice. Settling this question could be the basis of a *prediction* of low energy supersymmetry from string theory, or alternatively of large dimensions, with their distinctive consequences for future experiments.

Within the framework of supersymmetric approximate moduli, we have also discussed some cosmological issues. We have seen that the most often discussed problems of string cosmology might be solved if the dilaton – the field which fixes the ordinary gauge couplings – is fixed at a large scale, without breaking supersymmetry, while the other moduli sit at enhanced symmetry



points. Such a picture makes distinct low energy predictions. It predicts, as has been discussed elsewhere[27, 28] low energy supersymmetry *and* low energy breaking of supersymmetry.

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